High-Order Contact-Implicit Trajectory Optimization

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I. Introduction

Robots that dynamically make and break contact with their environment experience discontinuities in their state trajectories and dynamics. Traditional planning algorithms have modeled these discontinuities using a hybrid systems formulation, where each contact state is assigned a mode and impacts are handled via mode transitions. However, enumerating all possible modes for robots with many contact surfaces is computationally intractable.

In contrast, contact-implicit trajectory optimization methods have been used to generate contact sequences as part of the trajectory optimization problem without explicit enumeration of modes [1]. Unfortunately, these methods are currently limited to first-order integration accuracy, which negatively affects tracking performance [2]. We propose a new family of contact-implicit trajectory optimization methods that combine ideas from discrete mechanics and complementarity formulations of rigid-body contact to achieve any desired order of integration accuracy. Several simulated examples will be given using a third-order method.

II. Background

The classic time-stepping formulation of rigid-body contact proposed by Stewart and Trinkle [3] allows interpenetration constraints and Coulomb friction to be written as a set of linear complementarity conditions. At each timestep, an optimization problem is solved to determine contact impulses and propagate the system state forward to the next time step [4]. Posa et al. [1] incorporated these complementarity conditions into a direct trajectory optimization scheme to generate walking and manipulation trajectories without pre-specifying contact mode sequences.

A key feature of time-stepping methods is that they reason about integrals of forces over a time step, and therefore avoid technical issues associated with impulsive contact forces. *Variational integrators*, based on discretization of Hamilton's principle of least action and D'Alembert's principle of virtual work, also share this property [5]. It is therefore natural to combine them with the complementarity formulation of contact. This provides a unified and mathematically consistent framework for deriving time-stepping schemes of any desired order of accuracy.

III. Approach

Following [5], we begin with the integral form of the principle of virtual work:

$$\delta \int_0^T \mathcal{L}(q, \dot{q})\, dt + \int_0^T \boldsymbol{F}(q, \dot{q}) \cdot \delta q \, dt = 0. \qquad (1)$$

Taking variations at this stage results in the classical forced Euler-Lagrange equation. Instead, we break the integrals in (1) into smaller pieces,

$$\delta \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} \mathcal{L}(q, \dot{q})\, dt + \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} \boldsymbol{F}(q, \dot{q}) \cdot \delta q \, dt = 0, \qquad (2)$$

where $t_{k+1} = t_k + h$ and h is a fixed time step. Each smaller integral inside the summation is then approximated using a quadrature rule before taking variations. The result is a set of algebraic equations relating states at adjacent time steps. These algebraic equations form a variational integrator that can be used to simulate the system dynamics. Their order of accuracy is determined by the order of the quadrature rule used to approximate the integrals in (1).

We use quadratic polynomials to represent state and input trajectories over a time step, along with Simpson's rule, to produce a third-order variational integrator. We use the resulting set of algebraic equations as equality constraints to enforce dynamic feasibility in a direct trajectory optimization method. We then add complementarity conditions [4] as additional constraints to resolve contact impulses during each time step. The resulting algorithm is similar in spirit to [1], but achieves higher accuracy with a given number of knot points. Given its third-order integration accuracy, the new time-stepping scheme should provide performance on par with hybrid collocation methods [2] while being fully contact implicit.

References

[1] M. Posa, C. Cantu, and R. Tedrake, "A direct method for trajectory optimization of rigid bodies through contact," *International Journal of Robotics Research*, vol. 33, no. 1, pp. 69–81, Jan. 2014.

[2] M. Posa, S. Kuindersma, and R. Tedrake, "Optimization and stabilization of trajectories for constrained dynamical systems," in *Proceedings of the International Conference on Robotics and Automation (ICRA)*. Stockholm, Sweden: IEEE, 2016, pp. 1366–1373.

[3] D. E. Stewart and J. C. Trinkle, "An implicit time-stepping scheme for rigid body dynamics with inelastic collisions and coulomb friction," *International Journal for Numerical Methods in Engineering*, vol. 39, no. 15, pp. 2673–2691, 1996.

[4] M. Anitescu and F. A. Potra, "Formulating dynamic multi-rigid-body contact problems with friction as solvable linear complementarity problems," *Nonlinear Dynamics*, vol. 14, no. 3, pp. 231–247, 1997.

[5] J. E. Marsden and M. West, "Discrete Mechanics and Variational Integrators," *Acta Numerica*, vol. 10, pp. 357–514, 2001.